Stat 201:
Introduction to Statistics

Standard 20: Probability Distributions – Bell-Shaped Distribution (Z-scores)

Standard 21: Probability Distributions – Bell-Shaped Distribution (Percentiles)

Continuous Distributions!

- A continuous probability distribution has some sort of curve associated with it
  - We will concentrate on the normal distribution
  - Symmetric, bell shaped
  - Intro: https://www.youtube.com/watch?v=OWSOhpS00_s
- Remember the z-scores?
  - We use z-scores, the number of standard deviations, from the mean to look up the probabilities

Vocabulary

- A **model** is an equation, table, or graph used to describe reality.

- **The normal curve** is a model used to describe a continuous random variable that is said to be normally distributed
  - Symmetric about the mean
  - Bell Shaped

- A tour of the normal curve:
  - https://www.youtube.com/watch?v=772_n15Ke9Q

Let's Apply This to Continuous Variables

- **Numerically:** The possible values for a continuous random variable form an interval.
- There are infinitely many numbers on any interval, so the probability at any point is 0, so we look at the probability of intervals
  - Each interval has a probability between 0 and 1
  - The interval containing all possible values has probability equal to one

Let's Apply This to Continuous Variables

- **Graphically:** Continuous probability functions are called densities or distributions and look like smooth curves and the area under the curve represents the probability.

- The total area under the density is 1.

- Each observable value of the infinitely many has a line straight up to the density – this line has no area.

- An interval of observable values has a collection of lines that will make a shape – this shape has an area and gives the probability of the data on this interval.

Before the Bell Shaped Distribution

- The probabilities for the Continuous distribution are not as intuitive as they might be for the discrete case.

- To introduce this topic we will talk about a very simple distribution called the uniform distribution
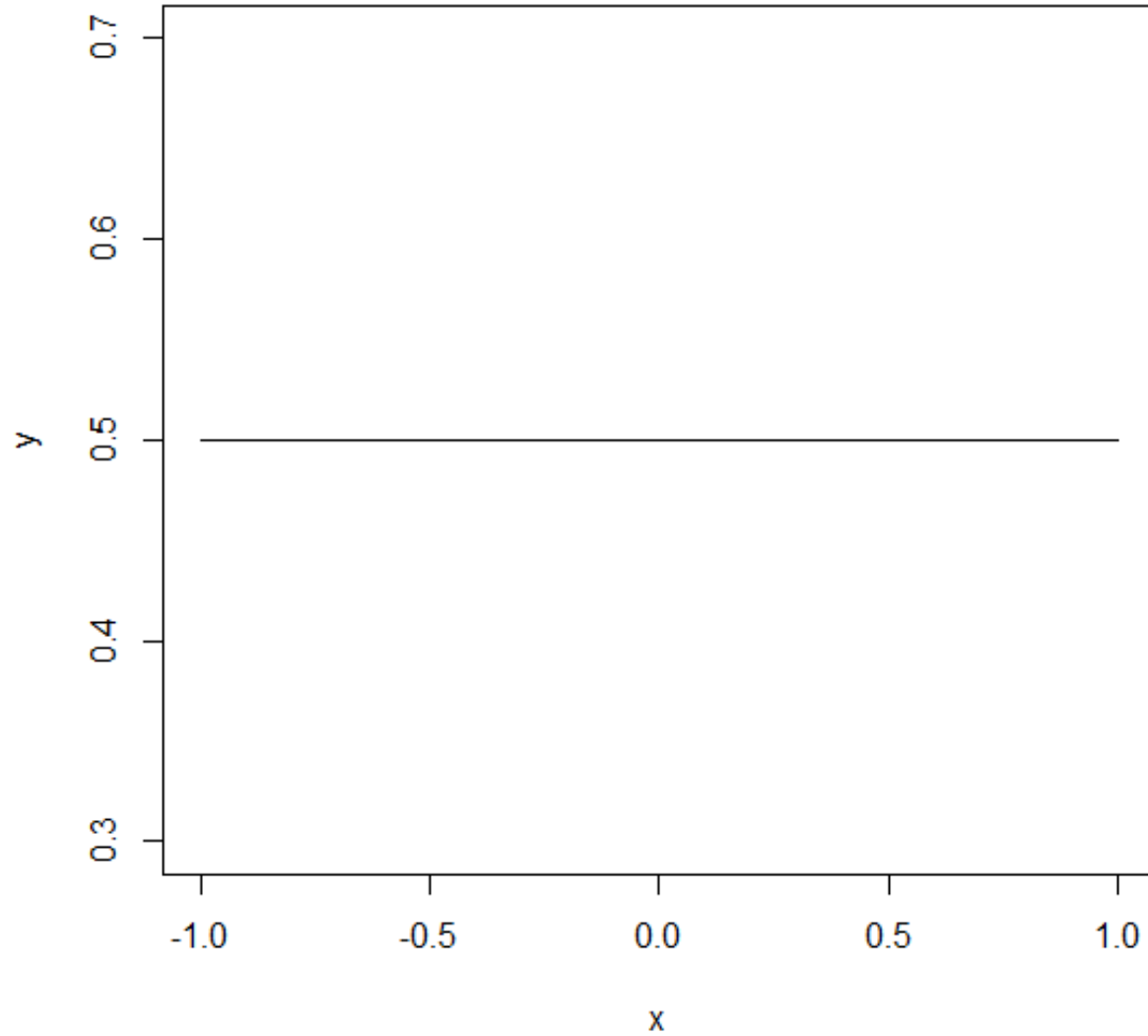
Before the Bell Shaped Distribution

- The probabilities for the Continuous distribution are not as intuitive as they might be for the discrete case.

- To introduce this topic we will talk about a very simple distribution called the uniform distribution

Before the Bell Shaped Distribution

- Consider the example where a random variable X could be any number between -1 and 1 with equal probability

- Instead of having a bell-shaped distribution we have a flat, uniform distribution that looks like a rectangle

# Before the Bell Shaped Distribution

Before the Bell Shaped Distribution

- Total probability under a continuous distribution is 1 – the rectangle has a width of 2 and a height of .5 so the area = 2*.5 =1

- The probability at any one point is zero – consider P(X=.25). X can be .25 only one way and there are infinitely many numbers between -1 and 1 so we have P(X=.25)= $\frac{1}{\infty}$ → 0

Before the Bell Shaped Distribution

- Recall that continuous variables are measured on an interval; their probabilities will be calculated the same way – on intervals

- The area under the distribution on the interval of interest is equal to the probability of X being on that interval
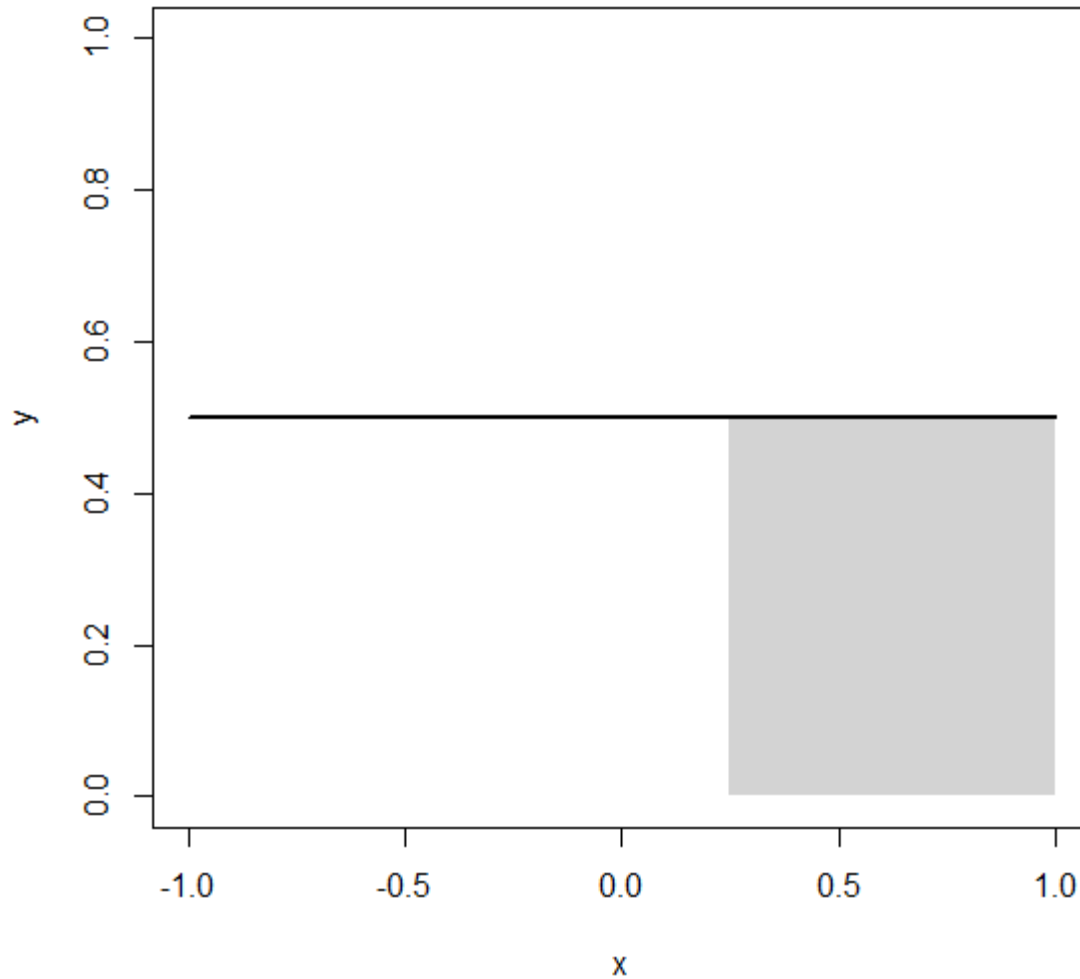
$$P(X \le .5) = punif(.5, -1, 1) = .75$$
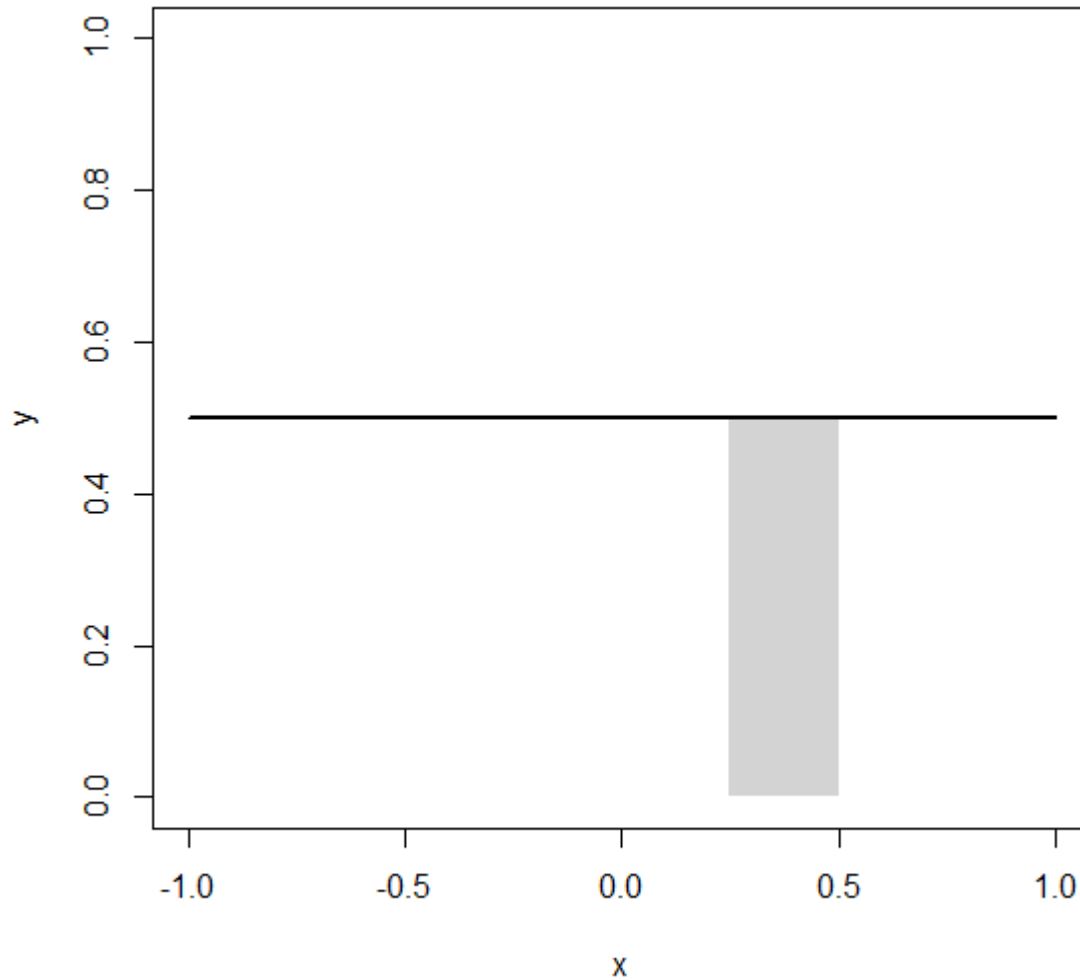
Note: Area $= .5 * 1.5 = .75$
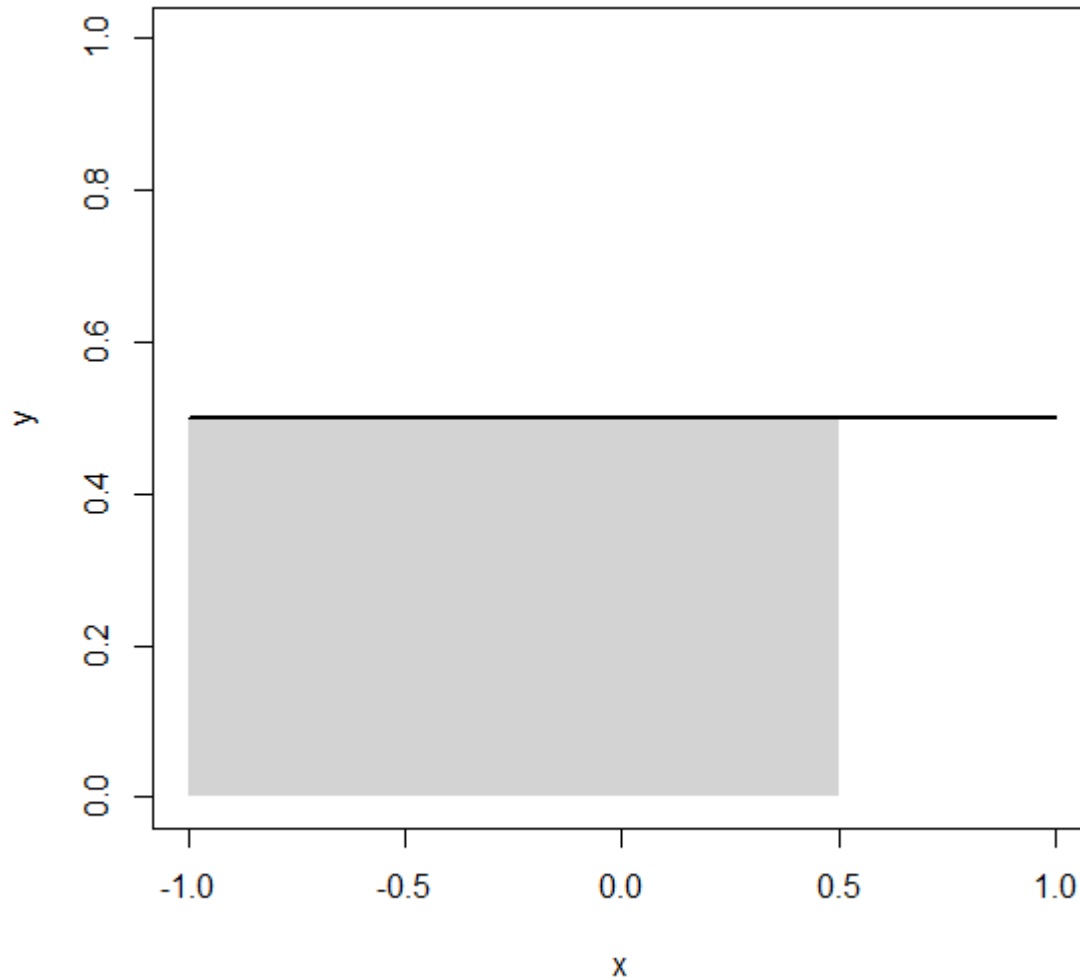
$$P(X \geq .25) = 1 - punif(.25, -1, 1) = .375$$

Note: Area $= .5 * .75 = .375$

$$P(.25 \leq X \leq .5) = \; punif(.5, -1, 1) \; - \; punif(.25, -1, 1) = .125$$

Note: Area $= .5 * .25 = .125$

The Normal Distribution

- The Normal distribution is used when a random variable X is 'normally distributed.' Many physical measurements follow this distribution.

- Density: $f_x(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{\left(\frac{-(x-\mu)^2}{2\sigma^2}\right)} I\{x \in \mathbb{R})$

- Probability X is on interval A=(a,b)

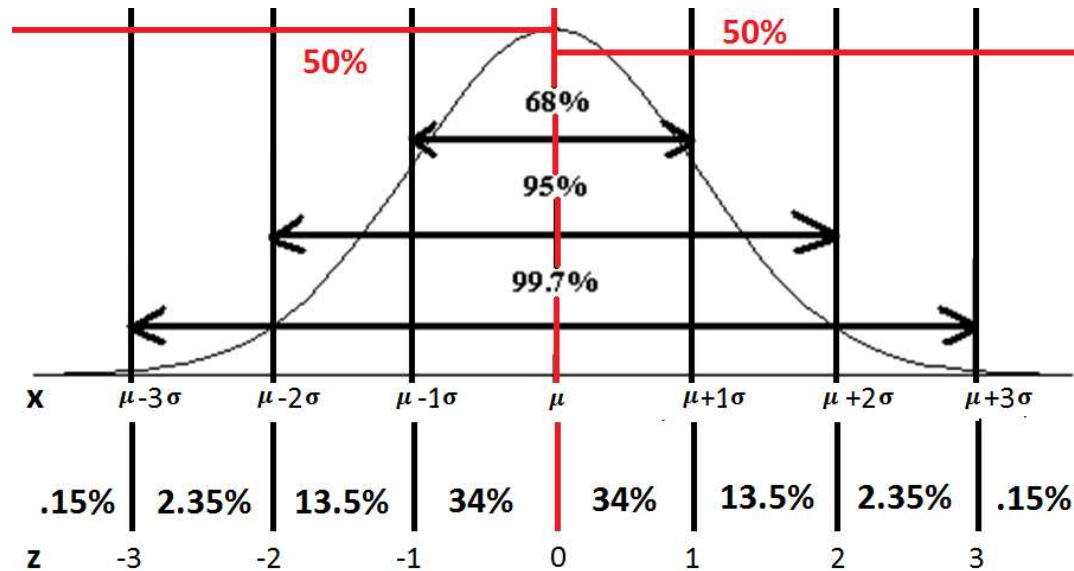$$P(X \in A) = \int_a^b f_x(x)dx$$

- Mean = μ
- Variance = σ^2

The Normal Distribution: Notation

- μ is the mean of the Normal random variable
- σ is the standard deviation of the Normal random variable

- X = the normal random variable
- X is the random variable, μ and σ are parameters; x will be the observation

Remember this?

- The total probability is one (100%)
- We'll use Z-scores to find the probability of other intervals not covered by the Empirical Rule – get excited!

Remember this?
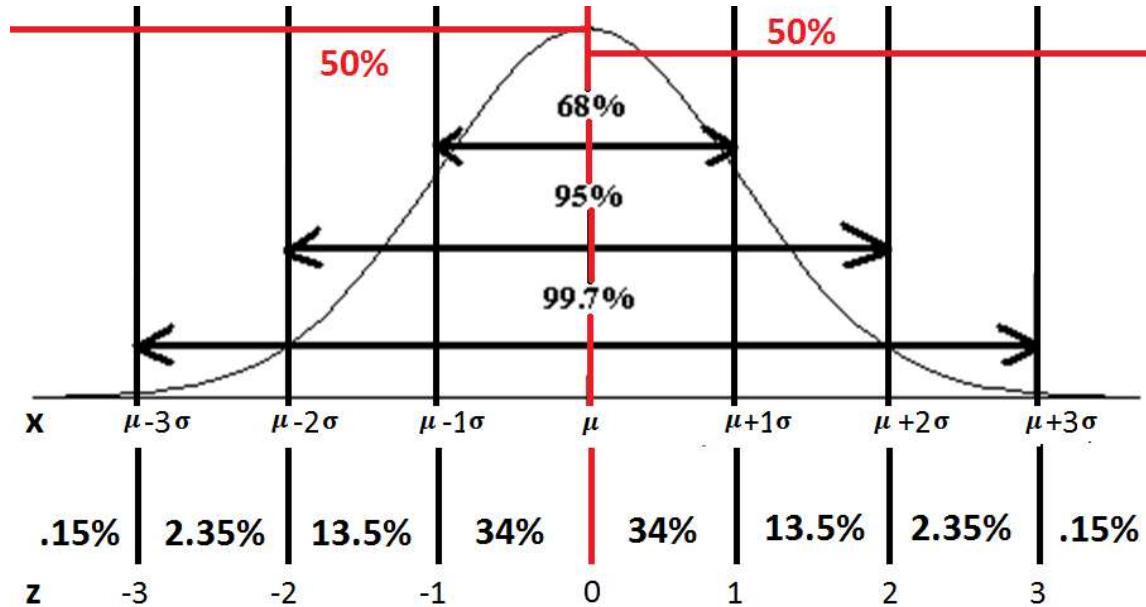
- **The area under the graph of a density function over an interval represents the probability of observing a value of the RV in that interval**

Normal Curve

- The normal curve is...
  - Bell-shaped
  - Symmetric about the mean
  - Follows the Empirical Rule
  - Not as easy as a rectangle to find the area
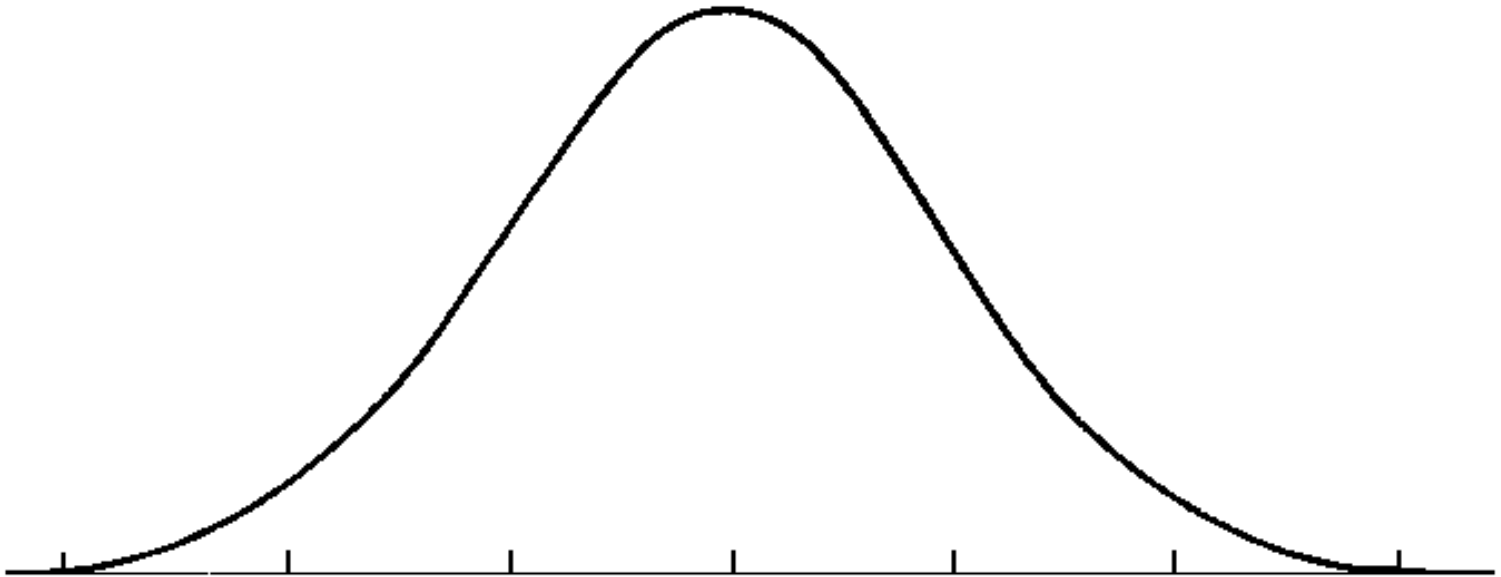
Normal Curve

- If we decrease the mean our normal curve will shift to the left

- If we increase the mean our normal curve will shift to the right

- If we decrease the standard deviation our normal curve will get more narrow

- If we increase the standard deviation our normal curve gets less narrow

Normal Curve

Legend:
- $\mu = 0, \sigma^2 = 0.2$ (red)
- $\mu = 0, \sigma^2 = 1.0$ (green)
- $\mu = 0, \sigma^2 = 5.0$ (blue)
- $\mu = -2, \sigma^2 = 0.5$ (purple)
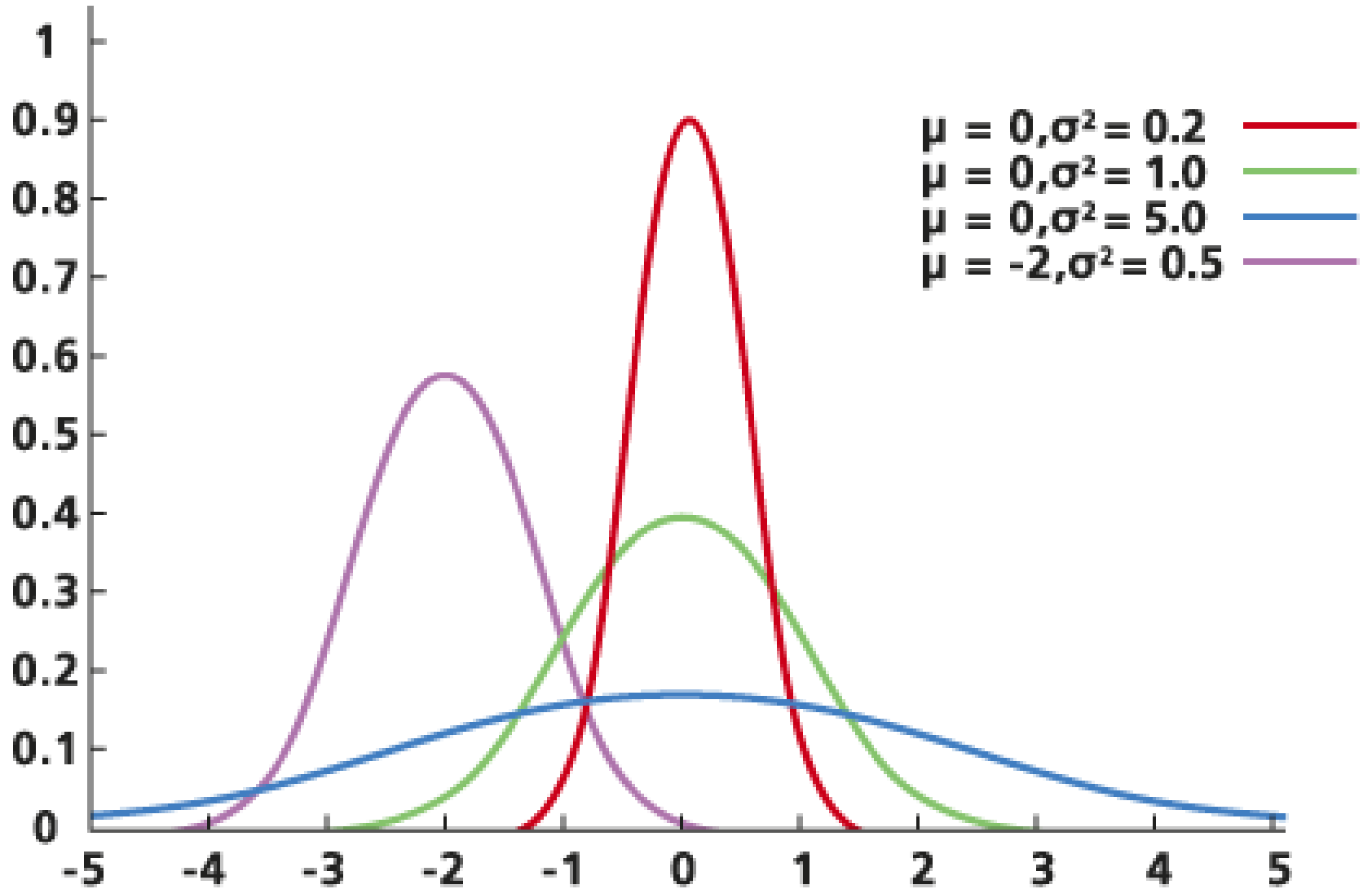
Properties of a Normal Curve

1. Symmetric around the mean
2. Highest point is at the mean=median=mode
3. Inflection points at $\mu \pm \sigma$
4. Total area under the curve is one
   - Area under the curve less/greater than the mean = .5
5. The graph approaches zero as we go out to either side
6. The Empirical Rule applies

Area Under a Normal Curve

- The area under a normal curve across any interval of values represents
  - The proportion of the population with the characteristic described
  - The probability that a randomly selected individual from the population will have the characteristic

Wrap Up the Normal Distribution

- We saved the best for last – the normal distribution is vastly important to statistics, particularly when we cover the central limit theorem

- In many problems going forward it is paramount to know whether or not our data is from a normal distribution

Is My Data Normal?

1. Look at a histogram or box plot – are they symmetric?
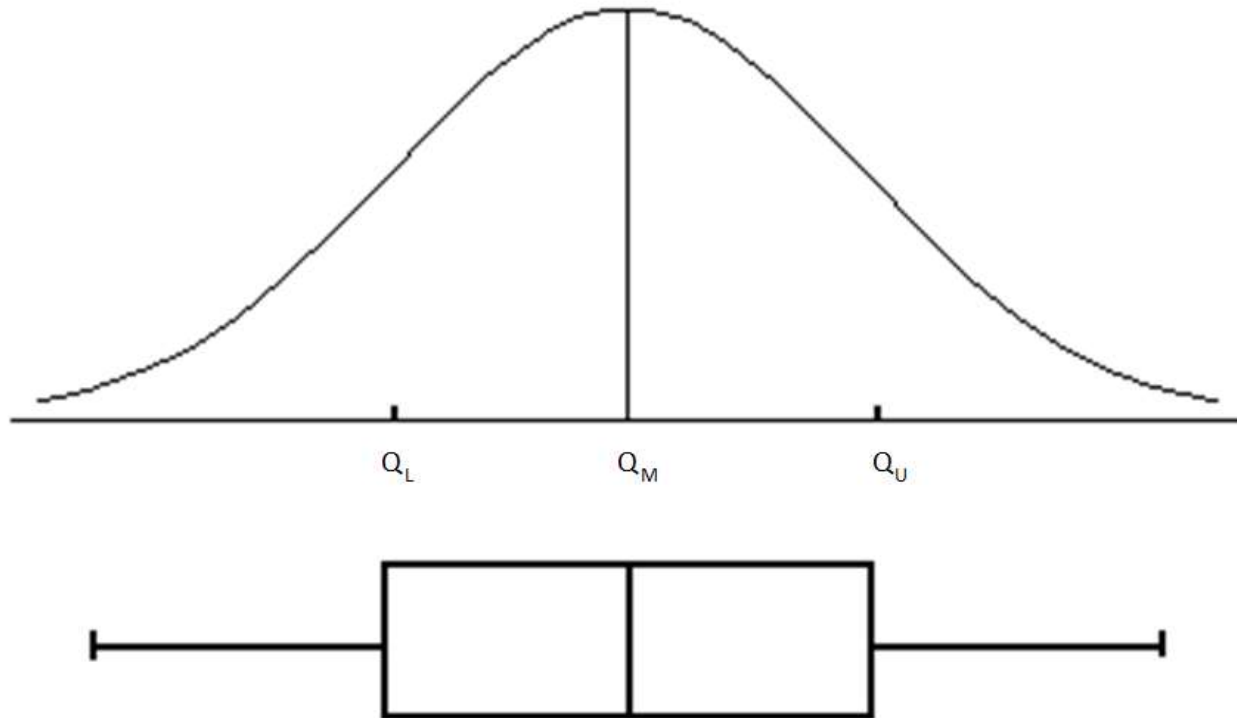
Is My Data Normal?

2. Do our sample intervals match the empirical rule?

- Are ~68% of the data between $\bar{x} \pm s$
- Are ~95% of the data between $\bar{x} \pm 2s$
- Are ~99.7% of the data between $\bar{x} \pm 3s$

Calculating Probabilities

- We have the Empirical Rule to find probabilities for certain points within one, two or three standard deviations from the mean, but what about the other points?

- It turns out that calculating these probabilities is very difficult and involves lots of Calculus without the trick I'm going to show you

Z transformation for Normal Calculations

- The trick is to transform our x's to z's, to transfer from the normal distribution of x to the N(0,1) **standard normal** distribution of z

- The Z score represents the number of standard deviations from the mean

$$z = \frac{observation - mean}{standard\ deviation} = \frac{x - \mu_x}{\sigma_x}$$

How x's and z's Line Up
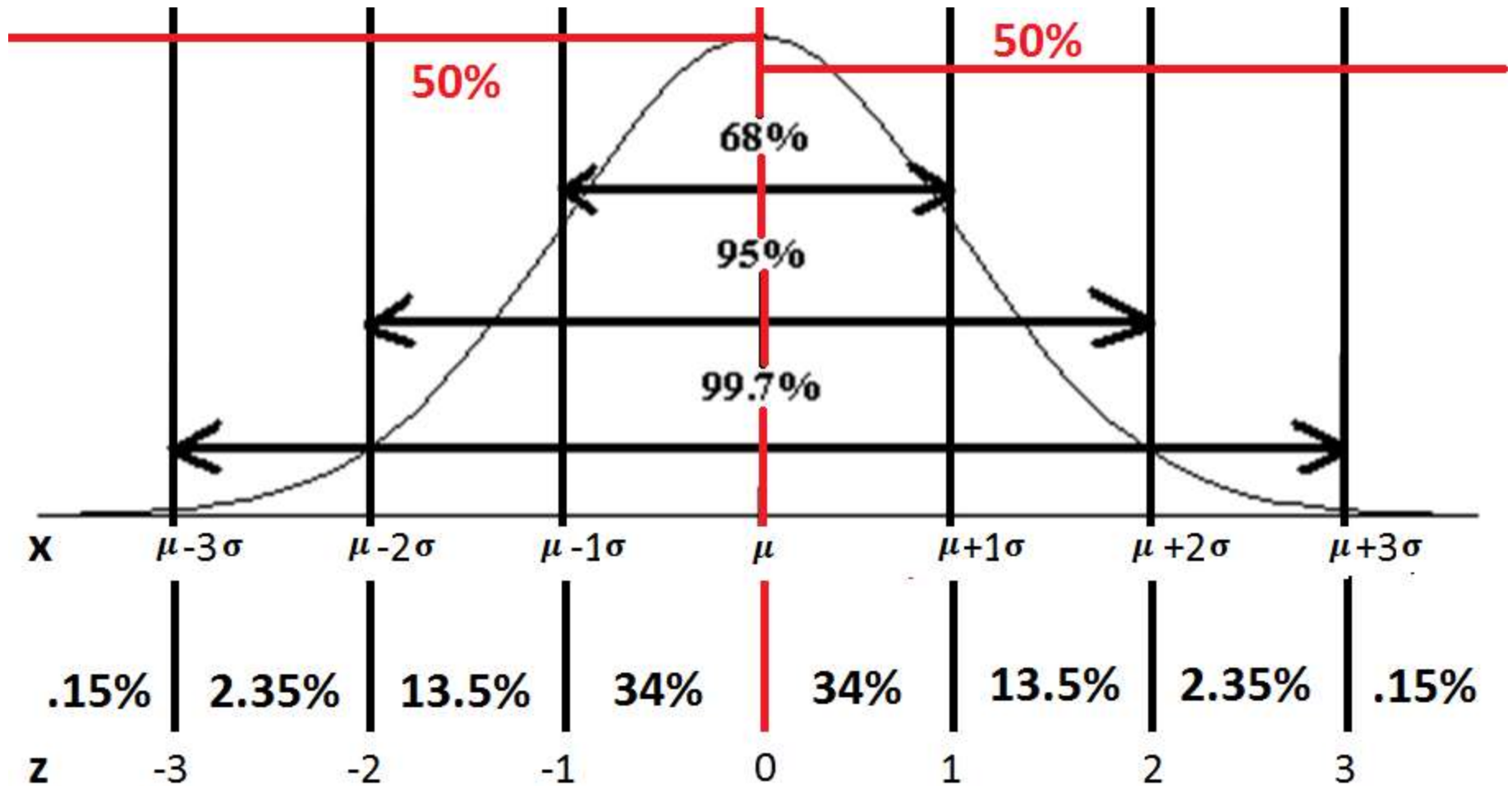
Calculating Probabilities

- So, in terms of z we have the Empirical Rule to find probabilities between points where z={-3,-2,-1,0,1,2,3}
  - 68% of the data lies between -1 and 1
  - 95% of the data lies between -2 and 2
  - 99.7% of the data lies between -3 and 3

Calculating Probabilities

- To figure out probabilities for points between these values we will look into a chart someone made for us that contains all the values in between that we would have to struggle with because they are very difficult and involve lots of Calculus

- Chart:

http://www.stat.ufl.edu/~athienit/Tables/Ztable.pdf

Calculating Probabilities

z	B .00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
A 0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
0.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852

- A and B tell us that the z-score is 0.40

  - A gives us the ones place and the tenths space (**0.4**0)

  - B gives us the hundredths place (0.4**0**)

Calculating Probabilities

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
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0.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852

- C tells us that the probability that we see an observation with a z-score of 0.40 or less is .6554
- The cross-hairs created when we look right of A and down from B gives us the less-than probability for that Z-score

Calculating Probabilities

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
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0.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852

- A and B tell us that the z-score is 0.25

    – A gives us the ones place and the tenths space (**0.2**0)

    – B gives us the hundredths place (0.2**5**)

Calculating Probabilities

z	.00	.01	.02	.03	.04	B .05	.06	.07	.08	.09
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
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0.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852

- C tells us that the probability that we see an observation with a z-score of 0.25 or less is .5987

- The cross-hairs created when we look right of A and down from B gives us the less-than probability for that Z-score

Z-Table

- $z = \dfrac{x - \mu_x}{\sigma_x}$

- Then using the z table we can then find:

  - $P(X < x) = P\left(Z < \dfrac{x - \mu_x}{\sigma_x}\right)$

  - $P(X \leq x) = P\left(Z < \dfrac{x - \mu_x}{\sigma_x}\right)$

  - $P(X > x) = 1 - P\left(Z < \dfrac{x - \mu_x}{\sigma_x}\right)$

  - $P(X \geq x) = 1 - P\left(Z < \dfrac{x - \mu_x}{\sigma_x}\right)$

Z-Table

- $z = \frac{x - \mu_x}{\sigma_x}$

- Then using the z table we can then find:

$$P(x_1 < X < x_2) = P(X < x_2) - P(X < x_1)$$

$$= P\left(Z < \frac{x_2 - \mu_x}{\sigma_x}\right) - P\left(Z < \frac{x_1 - \mu_x}{\sigma_x}\right)$$

Z-Table: Show the Empirical Rule

- The Empirical Rule states that 68% of the data lies between $\mu_x - \sigma_x$ and $\mu_x + \sigma_x$

- Let's pretend that we didn't know the Empirical Rule and find this probability using the z-table
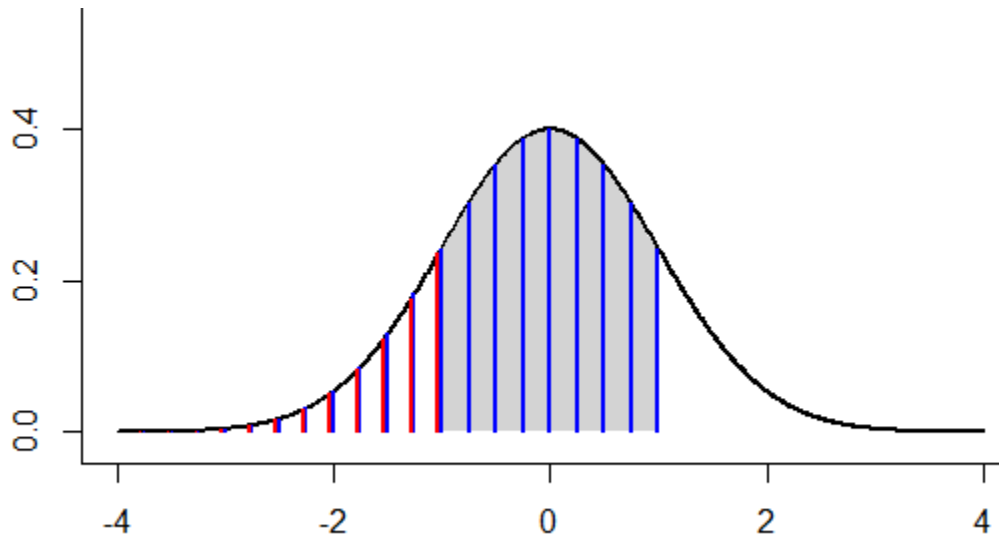
Z-Table: Show the Empirical Rule

1. Make sure the data you're talking about is normally distributed

   - This will be given in the problem

   - If not, you can look at a histogram of the data to see whether or not the histogram is symmetric and bell-shaped

Z-Table: Show the Empirical Rule

2. We can write the following by 'fitting pieces'(the blue take away the red)

• The grey shows the difference is what we want.

Z-Table: Show the Empirical Rule

3. Find the z-scores

$$z_{\mu-\sigma} = \frac{(\mu_x - \sigma_x) - \mu_x}{\sigma_x} = -1$$

$$z_{\mu+\sigma} = \frac{(\mu_x + \sigma_x) - \mu_x}{\sigma_x} = 1$$

Z-Table: Show the Empirical Rule

4. Find the percentiles by finding the crosshairs in the z-table

$$P(Z < 1) = .8413$$
$$P(Z < -1) = .1587$$

So,

$$P(\mu_x - \sigma_x < X < \mu_x + \sigma_x) = P(-1 < Z < 1)$$
$$= P(Z < 1) - P(Z < -1)$$
$$= .8413 - .1587 = .6826$$

Z-Table: Show the Empirical Rule

- The Empirical Rule states that 95% of the data lies between $\mu_x - 2\sigma_x$ and $\mu_x + 2\sigma_x$

- Let's pretend that we didn't know the Empirical Rule and find this probability using the z-table
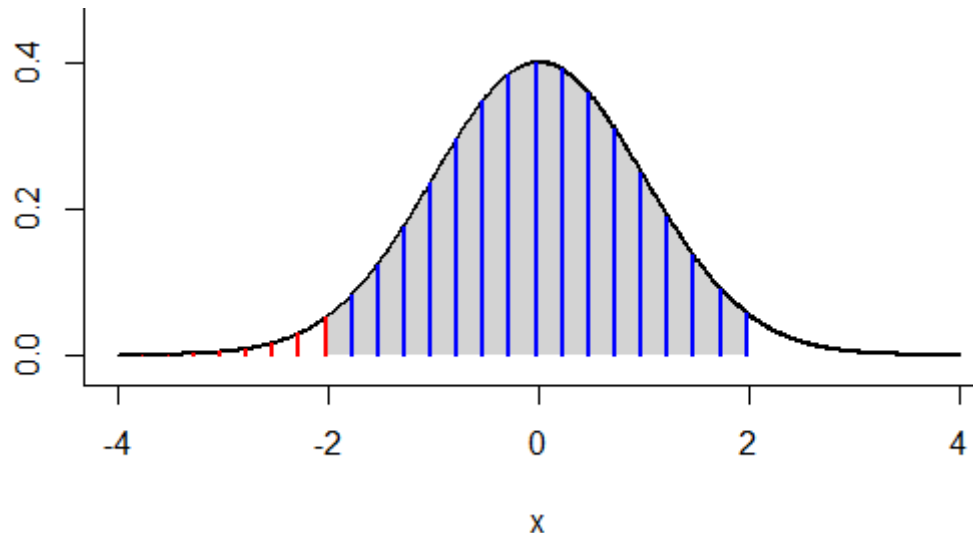
Z-Table: Show the Empirical Rule

1. Make sure the data you're talking about is normally distributed

    - This will be given in the problem
    - If not, you can look at a histogram of the data to see whether or not the histogram is symmetric and bell-shaped

Z-Table: Show the Empirical Rule

2. We can write the following by 'fitting pieces'(the blue take away the red)

- The grey shows the difference is what we want.

Z-Table: Show the Empirical Rule

3. Find the z-scores

$$z_{\mu-2\sigma} = \frac{(\mu_x - 2\sigma_x) - \mu_x}{\sigma_x} = -2$$

$$z_{\mu+2\sigma} = \frac{(\mu_x + 2\sigma_x) - \mu_x}{\sigma_x} = 2$$

Z-Table: Show the Empirical Rule

4. Find the percentiles by finding the crosshairs in the z-table

$$P(Z < 1) = .9772$$
$$P(Z < -1) = .0228$$

So,

$$P(\mu_x - 2\sigma_x < X < \mu_x + 2\sigma_x)$$
$$= P(-2 < Z < 2)$$
$$= P(Z < 2) - P(Z < -2)$$
$$= .9772 - .0228 = .9544$$

Z-Table: Show the Empirical Rule

- The Empirical Rule states that 99.7% of the data lies between $\mu_x - 3\sigma_x$ and $\mu_x + 3\sigma_x$

- Let's pretend that we didn't know the Empirical Rule and find this probability using the z-table
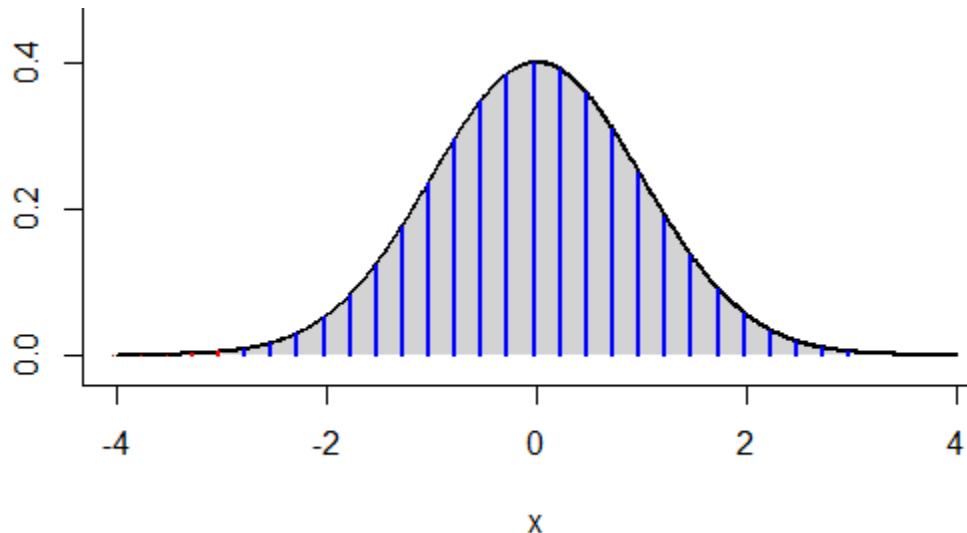
Z-Table: Show the Empirical Rule

1. Make sure the data you're talking about is normally distributed

   - This will be given in the problem
   - If not, you can look at a histogram of the data to see whether or not the histogram is symmetric and bell-shaped

Z-Table: Show the Empirical Rule

2. We can write the following by 'fitting pieces'(the blue take away the red)

- The grey shows the difference is what we want.

Z-Table: Show the Empirical Rule

3. Find the z-scores

$$z_{\mu-3\sigma} = \frac{(\mu_x - 3\sigma_x) - \mu_x}{\sigma_x} = -3$$

$$z_{\mu+3\sigma} = \frac{(\mu_x + 3\sigma_x) - \mu_x}{\sigma_x} = 3$$

Z-Table: Show the Empirical Rule

4. Find the percentiles by finding the crosshairs in the z-table

$$P(Z < 1) = .9987$$
$$P(Z < -1) = .0013$$

So,

$$\text{P}(\mu_x - 3\sigma_x < X < \mu_x + 3\sigma_x)$$
$$= P(-3 < Z < 3)$$
$$= P(Z < 3) - P(Z < -3)$$
$$= .9987 - .0013 = .9974$$
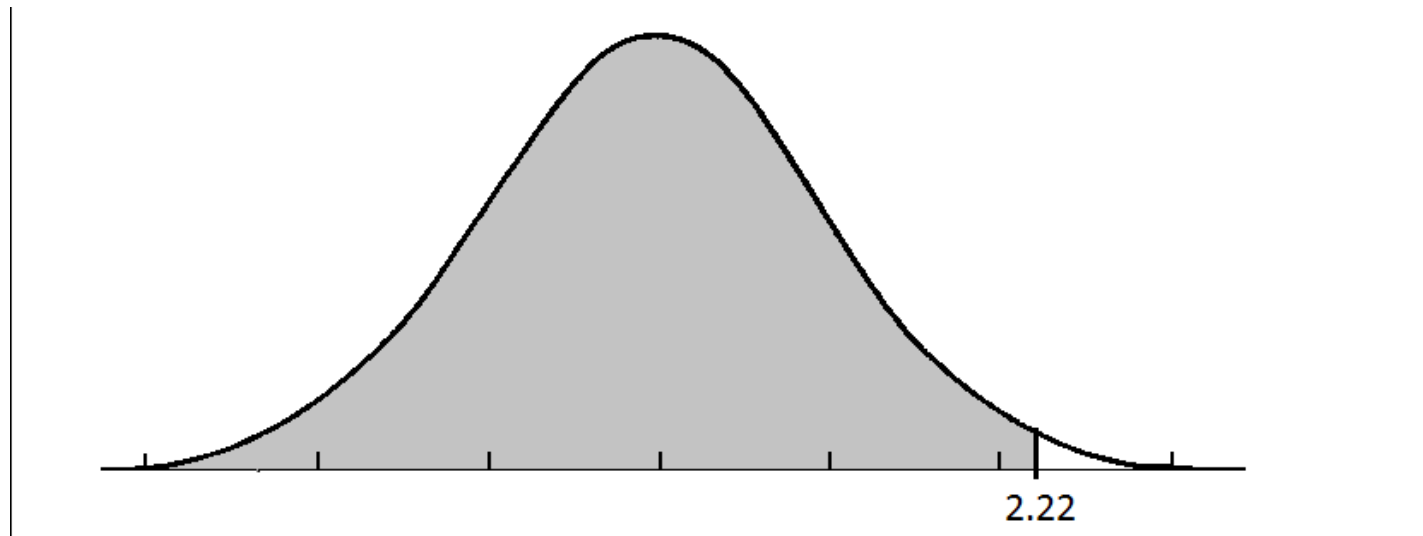
Z-Table Example 1

- Let's say you have a giant pet dragon and you decide you want to find out just how big he is compared to other dragons.

- So, you bring your dragon to the vet and after his check up the vet tells you that your pet dragon is 2.22 standard deviations above the mean in wingspan length, which is normally distributed, with a measurement of 397cm

Z-Table Example 1

- This is equivalent to saying your dragon's wingspan of x=397 gives him a z-score of 2.22

- Now that we know that x=397 gives z=2.22 we can find the percentage of dragons that are smaller than yours!

Z-Table Example 1

- Look up 2.22 in the z-table
  - This gives us P(Z<2.22) = .9868
- We now know that your dragon is bigger than 98.68 percent of all dragons, WOW!
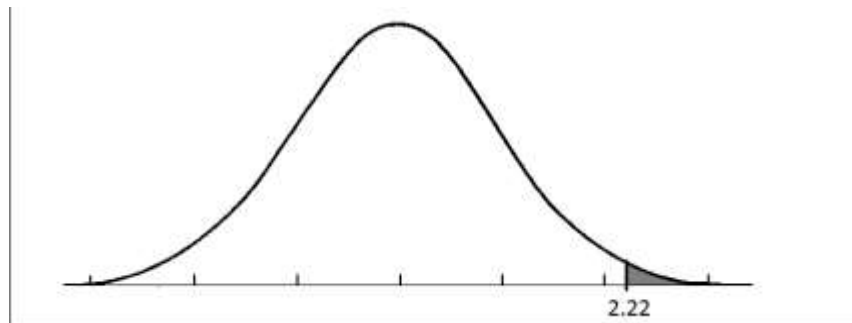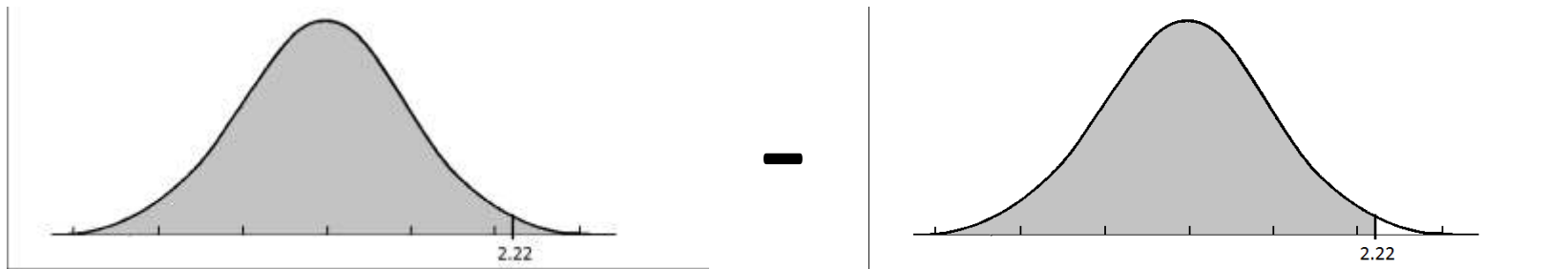
Z-Table Example 1

- What if we wanted to know the percentage of dragons that are bigger than yours?

- Here we would use complement rule because we cant look up P(Z>2.22) in the z-table.

- P(Z>2.22)=1-P(Z<2.22) = .0132

- We now know that only 1.32% of dragons are bigger than yours

Z-Table Example 1

- We can write the following by 'fitting pieces'
  - P(Z>2.22) = 1-P(Z<2.22)
  - The dark grey shows the difference is what we want.

Z-Table Examples 1

- This example could have played out very differently depending on what your vet told you, so let's take a look.

- So, you bring your dragon to the vet and after his check up the vet tells you, instead, that your pet dragon has a longer wingspan than 98.68% of dragons.

Z-Table Example 1

- Now, we just know that the percentile of your dragon is 98.68 but what if we wanted to know your dragon's actual wingspan?
- You do some research to find that dragon wingspans have a bell-shaped distribution with the following mean and standard deviation below
  - $\mu = 200$ cm
  - $\sigma = 88.\overline{738}$

Z-Table Example 1

1. First we need to find out which z-score gives us a percentile of 98.68.

   - Look into the chart to find .9868 or the value closest to it, here we can find .9868 at z=2.22

2. Set up the z-score equation $z = \frac{x-\mu}{\sigma}$ and solve.

   - $2.22 = \frac{x-200}{88.\overline{738}}$
   - $2.22 * 88.\overline{738} = x - 200$
   - $x = 2.22 * 88.\overline{738} + 200 = 397$ cm
   - Note: this matches the measurement the vet gave in the first scenario

Z-Table Example 2

- Now, let's consider just how hot your dragon is – don't get too excited, we're just curious about the temperature of the fire it breaths.

- So, before you leave the vet you ask about your dragon's fire temperature the vet tells you that your pet dragon is 1.48 standard deviations below the mean in fire temperature with a measurement of 400 c

Z-Table Example 2

- This is equivalent to saying your dragon's wingspan of x=400 gives him a z-score of -1.48

- Now that we know that x=400 gives z=-1.48 we can find the percentage of dragons who's fire is cooler than yours!

Z-Table Example 2

- Look up -1.48 in the Z table
  - This gives us P(Z<-1.48) = .0694 which is the probability we were looking for all along!

- Only 6.94% of dragons breath fire cooler than yours – looks like you have a pretty safe dragon
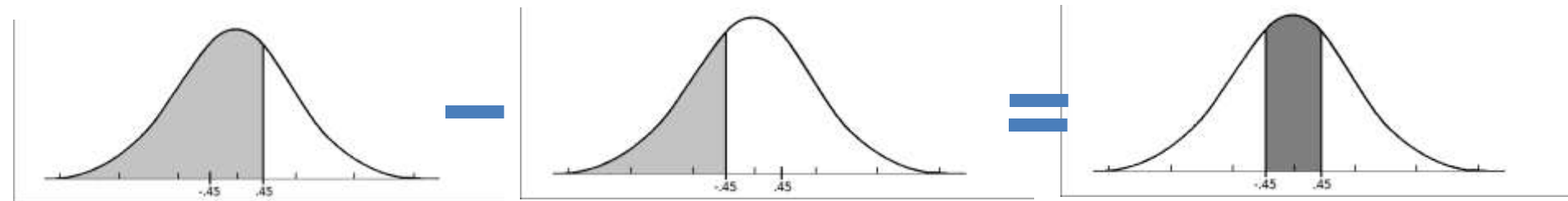
Z-Table Example 3

- Your dragon is adamant about getting elective surgery on their tail; to them it simply isn't long enough and it's worried about finding a mate to have little dragon babies with

- Your dragon expresses this desire with the vet and the vet tells your dragon that they simply don't need the surgery because their tail is about average in length – it's between .45 standard deviations of the mean tail length which is normally distributed

Z-Table Example 3

- You make it home with your disappointed dragon and you want to make it feel better by analyzing the statistics the vet gave

- After all, maybe the vet was right – what percentage of dragons have a comparable tail size?

Z-Table Example 3

- Within .45 standard deviations of the mean tells us that we have to worry about two z scores: z=-.45 and z=.45

- Let's consider the puzzle pieces we can fit here

Z-Table Example 3

- Within .45 standard deviations of the mean
    1) Find the probability of a z-score less than .45
        - P(Z<.45)=.6736
    2) 2) Find the probability of a z-score less than -.45
        - P(Z<-.45)=.3264

    3) This gives us P(-.45<Z<.45) = .6736-.3264 = .3472
        - This means that your dragon has a similar tail to 34.72% of dragons

Z-Table Example 3

- Even though your dragon now knows it has a similar tail to 34.72% of dragons it still doesn't feel very good about itself. It tells you to think of all the other dragons out there with giant tails – they'll be the dragons that get with Hollywood's dragons.

- Your dragon argues that the group of comparable dragons is too small by explaining that 65.28% of dragons aren't comparable

- You want to check its math

Z-Table Example 3

- Not having a comparable tail consists of being outside of .45 standard deviations of the mean

  1) Find what is within .45 standard deviations of the mean → P(-.45<Z<.45)

  2) Use complement rule → 1-P(-.45<Z<.45)

Z-Table Example 3

- Outside of .45 standard deviations of the mean
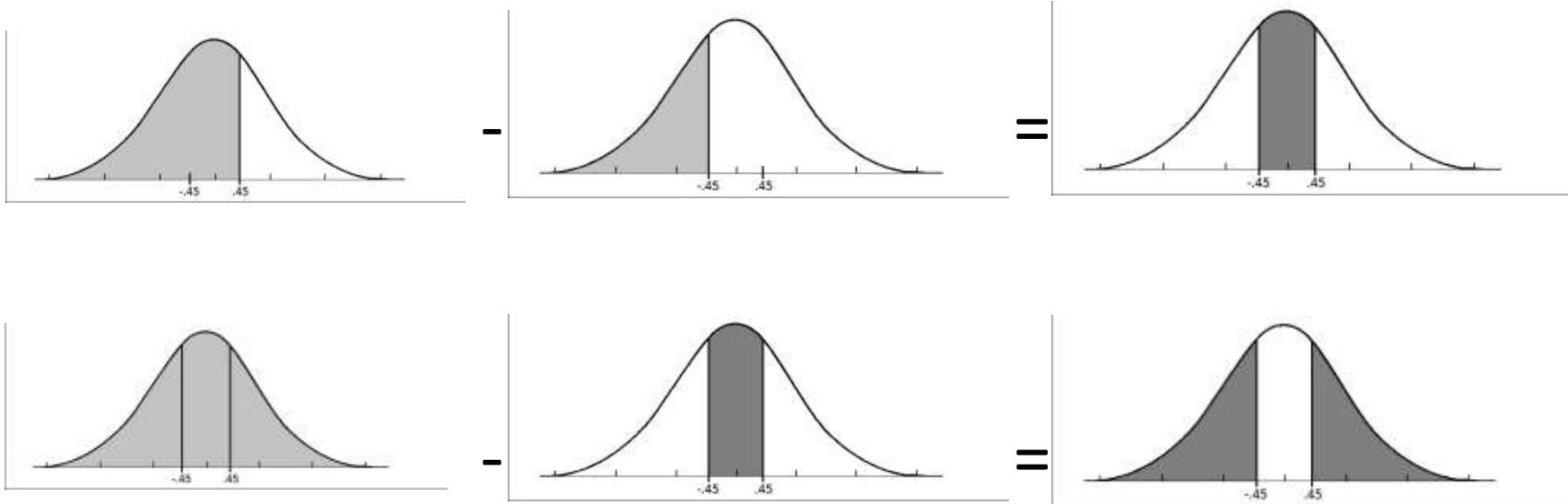
Z-Table Example 3

- Outside of .45 standard deviations of the mean
  1) Find what is within .45 standard deviations of the mean

     P(-.45<Z<.45)=P(Z<.45)-P(Z<-.45)=.6736-.3264=.3472

  2) Use complement rule

     1-P(-.45<Z<.45)=1-.3472=.6528

- Your dragon was right but you explain that not all dragons outside of it's group are bigger – some of the dragons in this group have smaller tails.

Z-Table Example 3

- Your dragon isn't receptive to any of the statistics. It doesn't matter what other dragons' tails looked like, it wasn't comfortable and wouldn't be until it had a tail in the top three percent of tails!

- You decide to let your dragon have the elective surgery to keep it happy – and if it got a Hollywood dragon you'd be set for life and their little dragon babies would be precious.

Z-Table Example 3

- How long does your dragons tail have to be to be in the top 3%?

- Find the Z-score of the observation where 3% of the data lies above it.
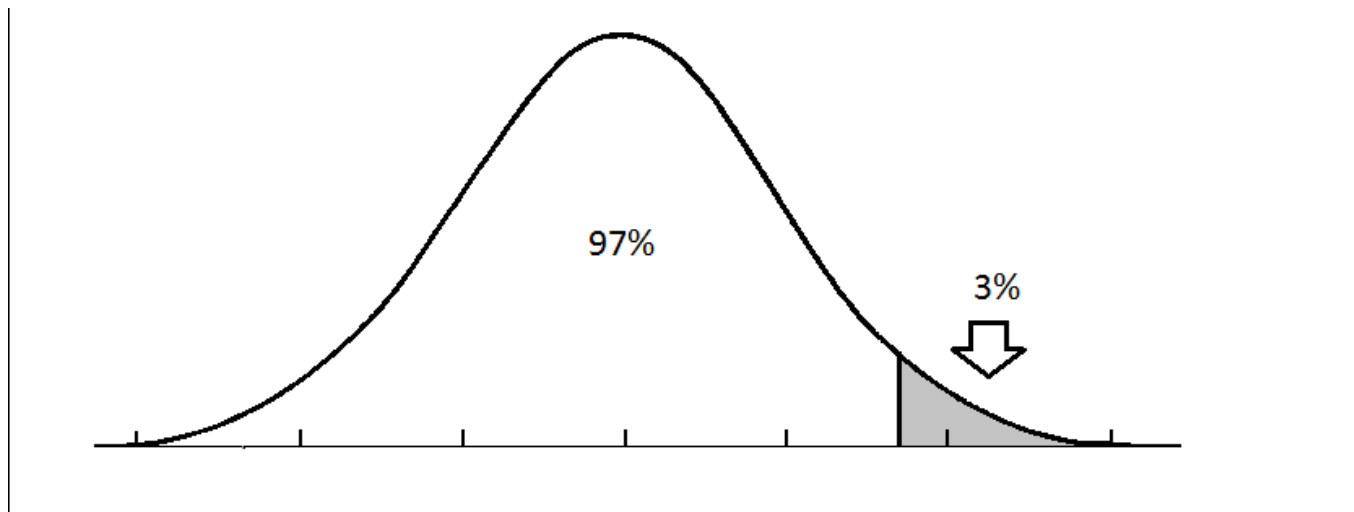
Z-Table Example 3
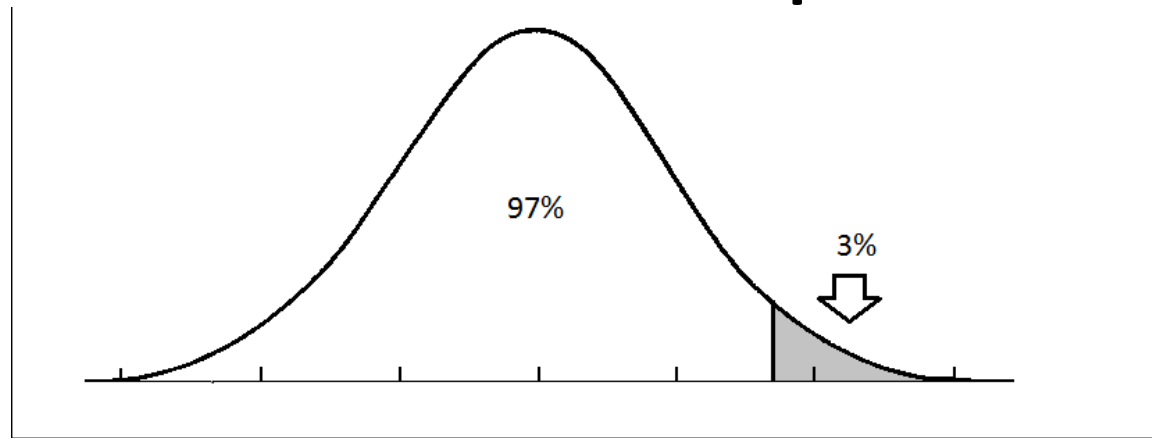
- We can only find percentiles in the z-table so we need to use complement rule to find out how much of the data lies below it (1-.03)=.97

- We need to find the z value that gives us .9700 or a value close to it. .9699 is the closest percentile in the table and that occurs when z=1.88

Z-Table Example 3
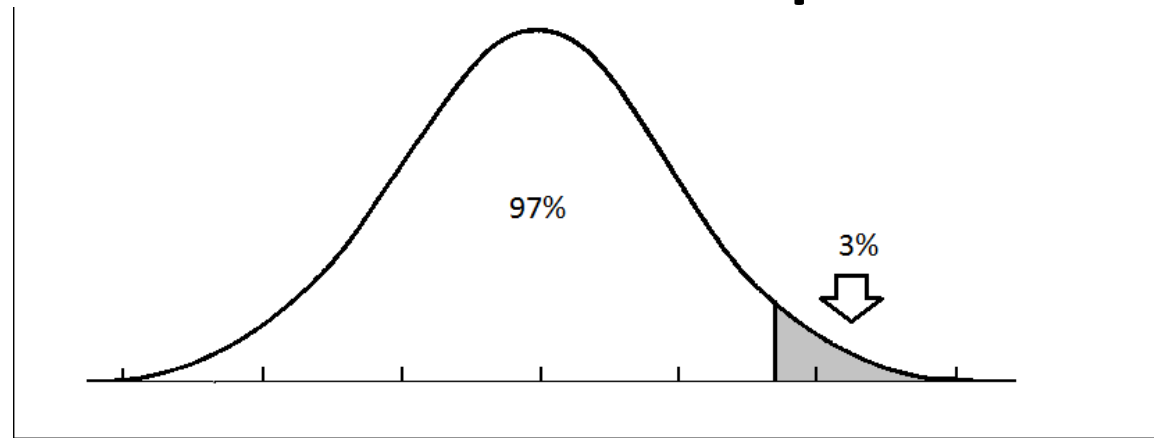
- With z=1.88 we can find the x observation that coincides with it so we can tell the plastic surgeon how long to make the dragon's tail

- To find the length from the z score you did some research to find that dragon tail length is normally distributed with the following mean and standard deviation…

  - $\mu = 244cm$
  - $\sigma = 25$

Z-Table Example 3
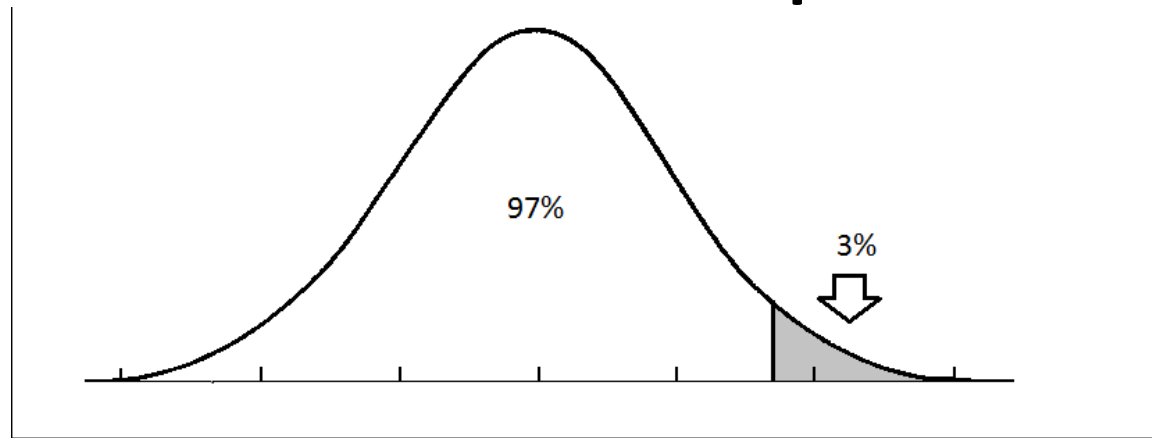
- Set up the z-score equation $z = \frac{x - \mu}{\sigma}$ and solve.

  - $1.88 = \frac{x - 244}{25}$
  - $1.88 * 25 = x - 244$
  - $x = 1.88 * 25 + 244 = 291$ cm

- To be in the top 3% your dragon's tail should be 291 cm long!

Z-Table Word Problem 1

- Wii Fit showed the average calories burned in 30 minutes was 111

- Assume a bell-shaped distribution with a standard deviation of 20

- Find the z-score of someone that burns 130 calories

$$z = \frac{observation - mean}{st.\,dev} = \frac{130 - 111}{20} = .95$$

Z-Table Word Problem 1

- Find the z-score of someone that burns 130 calories

$$z = \frac{observation - mean}{st.\,dev} = \frac{130 - 111}{20} = .95$$

- What is their percentile?

  – IT IS NOT .95 OR 95TH PERCENTILE!!!!

  – Look up .95 in the z table and that will give you the percentile

  P(Z<.95)=.8289 → 82.89th percentile

Z-Table Word Problem 1

- How many calories would a man have to burn to be at the 85th percentile?

  1) Find the Z that gives the 85th percentile

      .8508 is the closest value we can find within the table and

      z=1.04 is the z-value that corresponds to it.

  2) Set up and solve the following equation

    - $1.04 = \frac{x - 111}{20}$
    - x= $(1.04 * 20) + 111 = 131.8$

  3) Someone would have to burn 131.8 calories to be at the 85th percentile.

Z-Table Word Problem 2

- SAT score is bell shaped with mean score of 500 with a standard deviation of 100

- ACT score is bell shaped with mean score of 21 with a standard deviation of 4.1

- John scored 550 on the SAT and Jack 22.5 on the ACT – who had a better score?

$$z_{john} = \frac{550-500}{100} = .5 \quad \rightarrow \text{P(Z<.5)=.6915}$$

$$z_{jack} = \frac{22.5-21}{4.1} = .37 \quad \rightarrow \text{P(Z<.37)=.6443}$$

Z-Table Word Problem 2

- SAT score is bell shaped with mean score of 500 with a standard deviation of 100

- ACT score is bell shaped with mean score of 21 with a standard deviation of 4.1

$$z_{john} = \frac{550-500}{100} = .5 \qquad \rightarrow P(Z<.5)=.6915$$

$$z_{jack} = \frac{22.5-21}{4.1} = .37 \qquad \rightarrow P(Z<.37)=.6443$$

We can see that John's score means he did better than 69.15% of students and Jack did better than 64.43% of students. John did better.

Review

- We REQUIRE the data to be from a normal, bell shaped distribution

- We can calculate a z-score for an observation and find its percentile

  - X → Z → Percentile

- We can take the percentile, find the z-score and solve for the observation

  - Percentile → Z → X

Z-Table
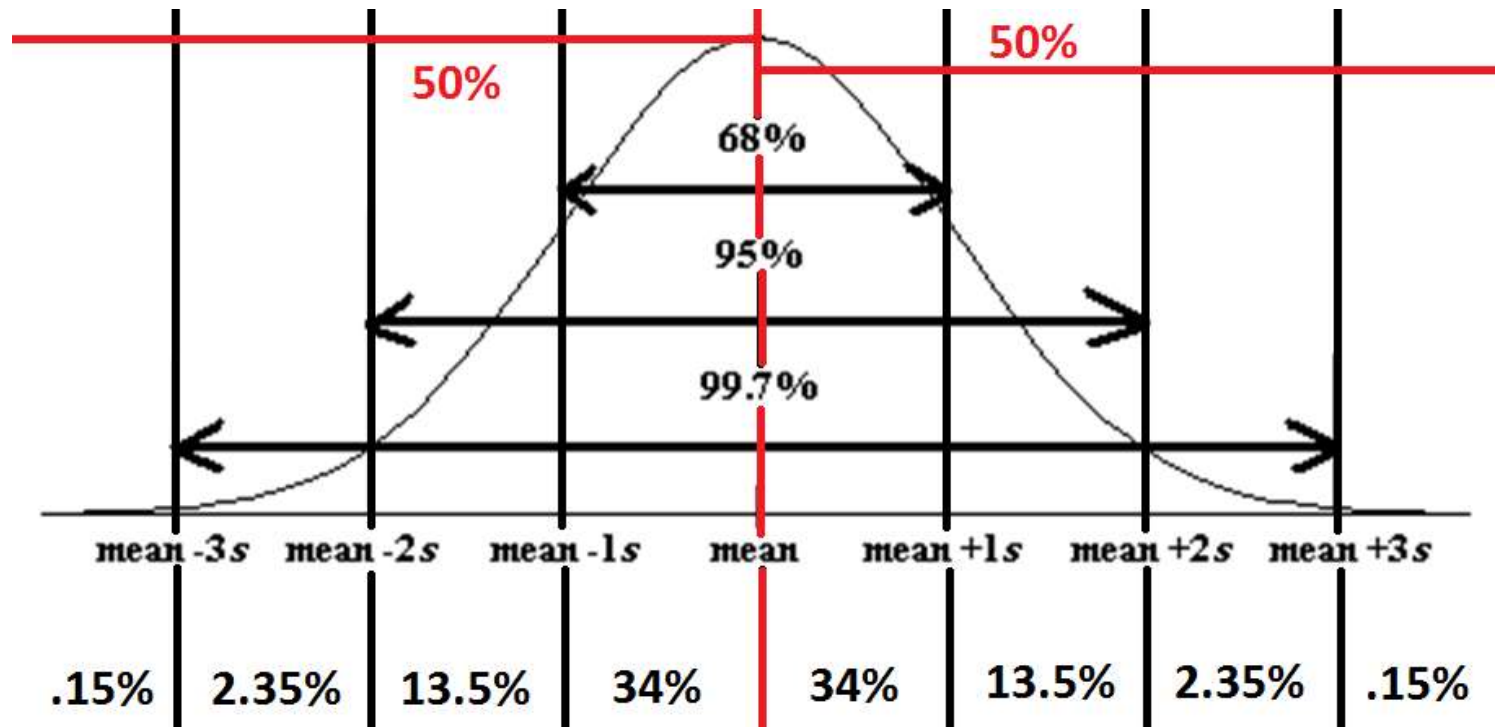
- Chart: http://www.stat.ufl.edu/~athienit/Tables/Ztable.pdf
- Walk-thorugh: https://www.youtube.com/watch?v=85G_PLBTX00
- TI83/84: https://www.youtube.com/watch?v=9N0F4IHong4
- TI83/84: https://www.youtube.com/watch?v=iuZNSXS44i8

Normal Probabilities (Two Sided) On your TI Calculator

- **<u>INPUT</u>**
  1. Press 2nd
  2. Press VARS
  3. Hit enter with '2: normalcdf(' highlighted
  4. Enter the lower bound, press ','
  5. Enter the upper bound, press ','
  6. Enter the mean, press ','
  7. Enter the standard deviation, press ','
  8. Press ')'
  9. Press ENTER

Normal Probabilities (Two Sided)
On your TI Calculator

- **<u>OUTPUT</u>**
  - This yields the probability of x being between the lower bound and upper bound
    - P(lower bound < X < upper bound)

  - This essentially computes the z-scores and finds the probabilities for you

Normal Probabilities (One Sided '<')
On your TI Calculator

- **<u>INPUT</u>**
  1. Press 2nd
  2. Press VARS
  3. Press enter with '2: normalcdf(' highlighted
  4. Press 1, press 2nd, press ',', press 99, press ','
  5. Enter the upper bound, press ','
  6. Enter the mean, press ','
  7. Enter the standard deviation, press ')'
  8. Press ENTER

Normal Probabilities (One Sided '<') On your TI Calculator

- **<u>OUTPUT</u>**
  - This yields the probability of x being less than the upper bound
    - P(X < upper bound)

  - This essentially computes the z-scores and finds the probabilities for you

Normal Probabilities (One Sided '>')
On your TI Calculator

- **<u>INPUT</u>**
  1. Press 2nd
  2. Press VARS
  3. Press enter with '2: normalcdf(' highlighted
  4. Enter the lower bound, press ','
  5. Press 1, press 2nd, press ',', press 99, press ','
  6. Enter the mean, press ','
  7. Enter the standard deviation, press ')'
  8. Press ENTER

Normal Probabilities (One Sided '>') On your TI Calculator

- **<u>OUTPUT</u>**
  - This yields the probability of x being greater than the lower bound
    - P(lower bound < X)

  - This essentially computes the z-scores and finds the probabilities for you

Normal Probabilities (Given Probability) On your TI Calculator

- **INPUT**
  1. Press 2nd
  2. Press VARS
  3. Press enter with '3: invNorm(' highlighted
  4. Enter the desired probability to the left, press ','
  5. Enter the mean, press ','
  6. Enter the standard deviation, press ')'
  7. Press ENTER

Normal Probabilities (One Sided '>')
On your TI Calculator

- **<u>OUTPUT</u>**
  - This yields the X that yields the desired percentile
    - P(X<Output) = the value in step four

  - This essentially computes finds the z-score with the correct percentile and solves the z-score equation for x for you
  - **NOTE:** This only gives the probability to the left, you will have to use complement rule to figure out other problems.

Normal Distribution on StatCrunch

- Here you don't have to calculate Z because StatCrunch does it for you!

- Open StatCrunch

- Stat→ Calculators→Normal→ Enter μ and σ→ insert whichever probability statement you need→Compute

Normal Distribution on StatCrunch

- If you are only given Z=z

- Open StatCrunch

- Stat→ Calculators→Normal→ Enter 0 for the mean and 1 for the std. Dev.→ insert whichever probability statement you need pretending X is Z→Compute

Continuous Distributions

The area under a normal curve across any interval represents	1. The proportion of the population with the characteristic described 2. The probability that a randomly selected individual from the population will have the characteristic
Z-Score	$$z = \frac{observation - mean}{standard\ deviation} = \frac{x - \mu_x}{\sigma_x}$$
P(X<x) =	$$P\left(Z < \frac{x - \mu_x}{\sigma_x}\right)$$
P(X>x) =	$$P\left(Z > \frac{x - \mu_x}{\sigma_x}\right) = 1 - P\left(Z < \frac{x - \mu_x}{\sigma_x}\right)$$

Calculating Probabilities

- To figure out probabilities for points between these values we will look into a chart someone made for us that contains all the values in between that we would have to struggle with because they are very difficult and involve lots of Calculus

- Chart:

http://www.stat.ufl.edu/~athienit/Tables/Ztable.pdf

Calculating Probabilities

z	B .00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
0.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852

- A and B tell us that the z-score is 0.40
  - A gives us the ones place and the tenths space (**0.4**0)
  - B gives us the hundredths place (0.4**0**)

Calculating Probabilities

z	B .00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
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observation with a z-score of 0.40 or less is .6554

- The cross-hairs created when we look right of A and down from B gives us the less-than probability for that Z-score

Calculating Probabilities

z	.00	.01	.02	.03	.04	**B** .05	.06	.07	.08	.09
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
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0.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852

- A and B tell us that the z-score is 0.25
  - A gives us the ones place and the tenths space (**0.2**0)
  - B gives us the hundredths place (0.2**5**)